

Design and Validation of the Methods for Comprehensive Characterization of the Hyperelastic Properties of Elastomers

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Doctoral Thesis Summary



Tomas Bata University

Faculty of Technology

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Design and Validation of the Methods for Comprehensive Characterization of the Hyperelastic Properties of Elastomers

**Návrh a validace metod pro komplexní charakterizaci
hyperelastických vlastností elastomerů**

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ABSTRACT

This thesis and the research work surrounding it, is oriented towards finding a solution to a problem in obtaining accurate material constants whenever only a single data set (i.e. uniaxial tension test data) is available in hyperelastic material characterization.

To begin with, the serious nature of the problem was highlighted through results of set of experiments. There, several material models were tried with two data fitting methods and the inaccuracy of data fitting with single data set could be proved beyond doubt from this exercise.

At the next stage, as a preliminary solution to the problem, a suggestion was given in the way of secondary data set generation from available data. The question at this point was about the method which could be adopted to generate the second data set. As an initial trial, exponential function was used with several exponents in order to generate data which could be consequently used as biaxial data. Amid some minor discrepancies, method delivered some promising results. Second approach was sorted in order to get a better trajectory for the generated biaxial data. In this method, initial uniaxial data set was divided in to two segments and each segment was differently addressed. As a result, the trajectory of generated data nearly resembled the real biaxial data. Data fitting preceded the data generation, provided very encouraging results too. However, method had some serious shortcomings such as, unit incompatibility, and lack of use of uniaxial data in the later half. Due to these reasons, the method was not further examined for the use in the work.

Final experiments were done with six materials. Base material and other constituents were different in each of these cases and it resulted in varied data distributions in both uniaxial and biaxial data. An exponential function was once again used with a different exponent in final tests. Proximity of generated data against real biaxial data was statistically tested. For the testing, 95% confidence interval was selected and most of the instances, generated data distribution was within the limit. Situations where, results differed, adjustment of confidence interval could be proposed with justification considering the hyperelastic material properties. Finally, Mooney-Rivlin model was used for data fitting as to further emphasize the results.

ABSTRACT (CZECH)

Dizertační práce a výzkum provedený v průběhu doktorského studia se zaměřuje na nalezení řešení v oblasti získání přesných materiálových konstant, v případě, že je k dispozici pouze omezený soubor dat k charakterizaci hyperelastických materiálů.

Na začátku práce je zdůrazněna závažnost výše zmíněného problému skrze výsledky experimentů. Datové soubory z testování hyperelastických materiálů byly vyhodnoceny na několika materiálových modelech za použití dvou různých metod pro určení hyperelastických konstant. Nepřesnost určení konstant při využití dat pouze z měření jednoosého tahu byla jasně prokázána. V další fázi výzkumu, bylo navrženo předběžné řešení tohoto problému, a to ve formě generování druhého souboru dat (dvouosý tah) z dostupných dat pro jednoosý tah. Předmětem výzkumu tedy bylo stanovení vhodné metody pro generování druhého datového souboru. Pro prvotní testování byla pro toto generování zvolena exponenciální funkce.

Mimo drobné nesrovnalosti, byly výsledky této metody slibné. Dalším krokem řešení bylo v nalezení přesnější funkce pro generovaná biaxiálních dat. V rámci této metody se křivka pro dvouosý tah rozdělila na dva segmenty, přičemž každý segment byl řešen odděleně. Byla získána data, která blízce připomínala skutečný biaxiální datový soubor. Avšak tato metoda vykazovala vážné nedostatky, jako je například nekompatibilita jednotek generovaných dat a nedostatečný počet dat v druhém segmentu. Z těchto důvodů nebyla tato metoda dále použita. Finální experimenty byly provedeny se šesti různými elastomery. Ty se lišily základním materiálem kaučukové směsi a dalšími složkami, což se projevilo v různorodosti jednoosých i dvouosých dat. Shoda generovaných dat se skutečným dvouosým tahem, byla statisticky testována. Pro testování, byl zvolen interval spolehlivosti 95 %, a ve většině případů, byla shoda potvrzena. Pro situace, ve kterých se výsledky lišily, bylo navrženo upravení intervalu spolehlivosti, což bylo odůvodněno hyperelastickými vlastnostmi materiálů. V závěru práce je přínos výsledků ověřen při určení materiálových konstant pro Mooney-Rivlinův hyperelastický model.

1. INTRODUCTION

Materials that exhibit large elastic strains at relatively moderate loads are called hyperelastic materials [1-4]. There are many materials in this category and applications are also many [5]. At the same time these materials show some outstanding properties [6-10]. However, due to nonlinear behaviour, mechanical characterization of them is difficult. As these materials are complicated to analyze, process is done in stages [11-13].

When it comes to mechanical characterization of rubber like materials, the task is challenging due to nonlinear behaviour. Characterization is usually done through set of pre-selected material models. There are many as forty models to select with and selection is done after consideration of application and few other factors. Consequently, material constants are obtained through data fitting.

Fitting is done using data collected through laboratory experiments. Due to difficulties in obtaining data with several deformation modes, only uniaxial test data is often used for fitting. However, method proved to be inaccurate. Therefore, with this work, it is expected to find a method to get an accurate material constants in such instances.

The very first hyperelastic material model was introduced by Melvin Mooney as a general strain energy function in 1940 [14] and data required for the testing of models was then provided by Treloar [15]. In 1948, Rivlin improved the first model, and it came in to existence as Mooney-Rivlin model [16]. This is the most frequently used model for the elastomer characterization. The simplest model of all, the Neo Hookean model [16], is a special case of the two parameters Mooney-Rivlin model. Further to these initial models, in 1967, Valanis and Landel [17], introduced a new method of representing the strain energy function. There are few common models developed later on by Ogden, Yeoh [18], Arruda and Boyce [19]. There are many other models which are less known and could be used in specific applications [20-27]. There are many researchers who contributed to this particular field and some of their works are specifically related to present work

here [28-50]. Furthermore, material testing methods in this field also were examined extensively [51-68] in order to improve the standard of experiments.

2. THE OBJECTIVE

As so far mentioned, hyperelastic material characterization leaped along several fronts over the last eighty or so years. Numerous models came in to existence and new methods of analysis were tried. In addition to that, advanced methods for testing of different strain modes were also established. Development of powerful computers and introduction of FEA tools also further simplified the hyperelastic material characterization.

However, amid all these new developments in the hyperelastic material research area, the problem of elimination of complicated laboratory tests such as eqi-biaxial testing for data collection, seems yet to be addressed. On the other hand, due to such complexities and cost concerns a single data set, i.e. uniaxial data, is frequently used. Amid repetitive use, method known to be erroneous. Therefore an investigation for an alternative solution to address this issue became a necessity and could be well justified.

2.1 The Aim

Having mentioned the necessity, we could clearly outlined the aim or the objective of the work as follows. That is to find a method of obtaining realistic and accurate material constants whenever only uniaxial data is available.

2.2 The Solution

Throughout this research work, possibilities of replacing data obtained through complicated and sometimes inaccurate biaxial experiments, by set of artificial data generated through uniaxial data is examined.

2.3 The Approach

First of all, uniaxial data set is obtained through typical standard test. Consequently, data set thus obtained is manipulated through a mathematical formula in such a way that, second set of data could be obtained. Thereafter, this second set of data, which could be considered as an alternative to the missing biaxial data could be used for the combined data fitting together with uniaxial data.

3. STATE OF THE ART

3.1 The Chemistry of Rubber

Rubber is, in simple terms, a material that can be stretched as much as twice or more of its original size and still could be formed in to its initial shape once released. The structure gives the rubbery effect to the material. Chemically rubber is a hydrocarbon and its main constituent is polyisoprene. Typical appearance of these molecules is depicted in figure 3.1.

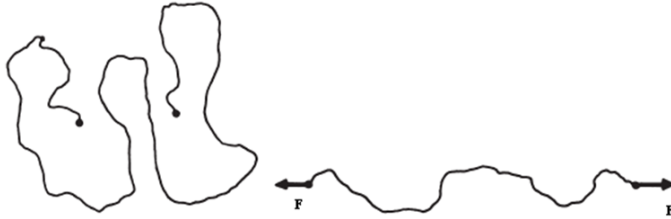


Fig. 3.1 Chain like structure of rubber [40]

To improve the rubber material strength and to transform it to useful engineering material, during the early ages of the development, a process called vulcanization was introduced.

The Vulcanizing Process

In this process, the long chain molecules of rubber materials are cross-linked through added foreign material at elevated temperature (140° – 180° C) as shown in figure 3.2.



Fig. 3.2 Rubber vulcanization process [40]

3.2 General Purpose Elastomers

Elastomers or rubberlike materials that are found in the industry can be broadly categorize in to two groups as general and special purpose elastomers. Bulk of rubber products manufactured today fall in to the group of general purpose elastomers. They comprised mostly, natural Rubber (NR), polyisoprene (IR), polybutadiene (BR), styrene-butadiene (SBR), nitrile-butadiene (NBR) and ethylene propylene rubber (EPR / EPDM). These elastomers are often used because of their good physical properties, processability and adoptability. In addition to that, they are economical too. Though there are many positives, some

negative properties are also there. Less heat, oil and solvent resistant are dominant in negative side of these materials. Besides, some of them are susceptible to ozone and oxygen attacks too [43].

3.3 Additives and Compounds

In the industry of rubber, there are many diverse applications as we already mentioned. These applications demand specific properties from rubber materials. Base materials alone cannot provide such exact properties. Therefore, they are mixed with various additives to achieve desirable effects.

Carbon Black, Silica and Talc are the main additives in rubber making process. Besides, there are some other minor adding agents used at the rubber mixing stage such as oils, wax, and fatty acids for process improvement and pigments for aesthetics and colour.

3.4 Mathematical Models

When one needs to select a hyperelastic model for mechanical characterization of a particular elastomer, there is a large group of modal forms to select from. However, it is established that the selection of model depends on factors such as material application, corresponding variables and available data [8]. Four major qualities of good material model are identified [9].

From these models, Neo-Hookean model, Mooney-Rivlin model, Yeoh, Arruda-Boyce model and Ogden model are more prominent.

3.5 Curve Theory

The main task of this work is to obtain a data set which matches a distribution similar to the typical biaxial data distribution. Uniaxial data set is used to obtain this biaxial data distribution. Therefore, in order to get a one distribution from the other, it is vital to study mathematical options available for this task.

Power function is a one such function that gives a specific shaped curve in the x-y domain. It can be given as in Eq. 3.1

$$y = x^n \tag{3.1}$$

Then, there is the general and natural exponential functions. They can be given as in eq. 3.2 and 3.3 respectively.

$$y = b^x \tag{3.2}$$

$$y = e^x \tag{3.3}$$

Other than these general curves, there are certain manipulations that can be done on general curves to get specific effects on them. Transitions, Enlargement and contraction, Reflection can be considered as common such manipulations.

3.6 Statistical Tools

Statistical tools are normally used to evaluate raw data obtained through laboratory experiments to get a meaningful results for further analysis. In this particular case, data obtained through three basic tests are fitted in to a predetermined model using non-linear regression technique.

3.7 Digital Image Correlation (DIC)

Metrology is one major corner stone of any research. It provides the data required for evaluation part to the research work. For hyperelastic material characterization, this involves in basically results of three tests. Namely these tests are, uniaxial, biaxial and pure shear. In order to get accurate results Digital Image Correlation (DIC) is used in our work.

Digital Image Correlation is a non-contact optical strain measuring technique. The measuring system comes complete with a digital camera, zoom objective and PC software.

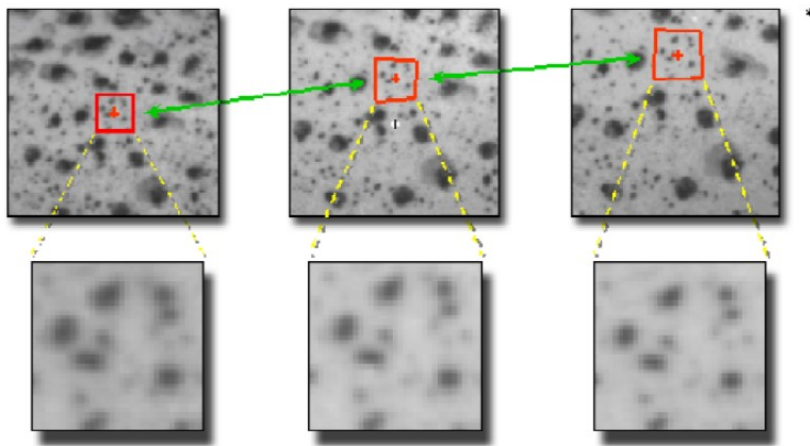


Fig. 3.3 Migration of subset due to deformation

In DIC, a shift in image pixel position is tracked through series of images when deformation is taking place due to the applied load on specimen during the test (Fig. 3.19).

4. RESULTS & THE DISCUSSION

This chapter presents the results of experiments carried out during the full stretch of the work. Apart from final set of experiments, three other experiments were done related to this work. These initial experiments were done using the data previously obtained. At the end, final experiments were done in order to test the proposed solution and reach ultimate objectives of the research.

4.1 Presentation of Problem (Exp. -1)

As a starting point to the research work, the risk of using only single data set, i.e. uniaxial data for fitting, in general to most hyperelastic material models, and in particular to Mooney-Rivlin model was established with scientific evidence. In this effort, a detailed comparison was done related to Mooney-Rivlin two, parameters, Mooney-Rivlin three parameters and Yeoh models.

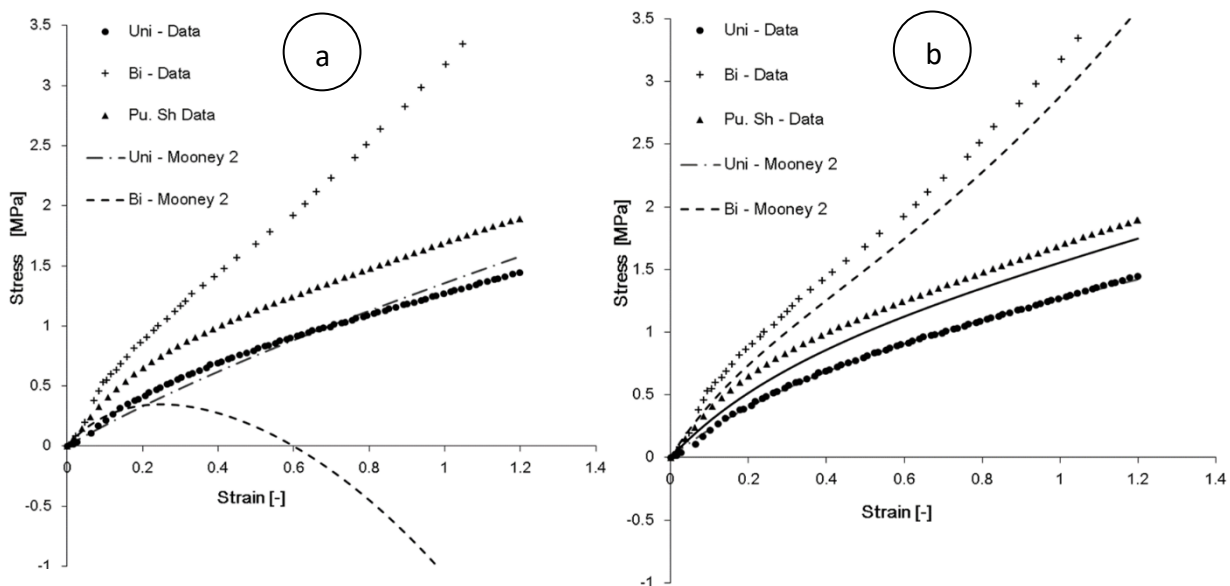


Fig. 4.1 Mooney two p. model comparison (a-only uniaxial, b-combined)

Figure 4.1 to - figure 4.3 show resultant curves obtained for this analysis. Three models tested for single and combined data fitting showed mixed results. Mooney-2 seems the most improved due to the combined data fitting. Mooney-3 model could be considered partially improved with the multiple data set fitting, while Yeoh model seems not responsive to change in data fitting technique. However, discrepancies in all three models related to only uniaxial data fitted curves prove that method is not accurate and therefore less suitable for the mechanical characterization.

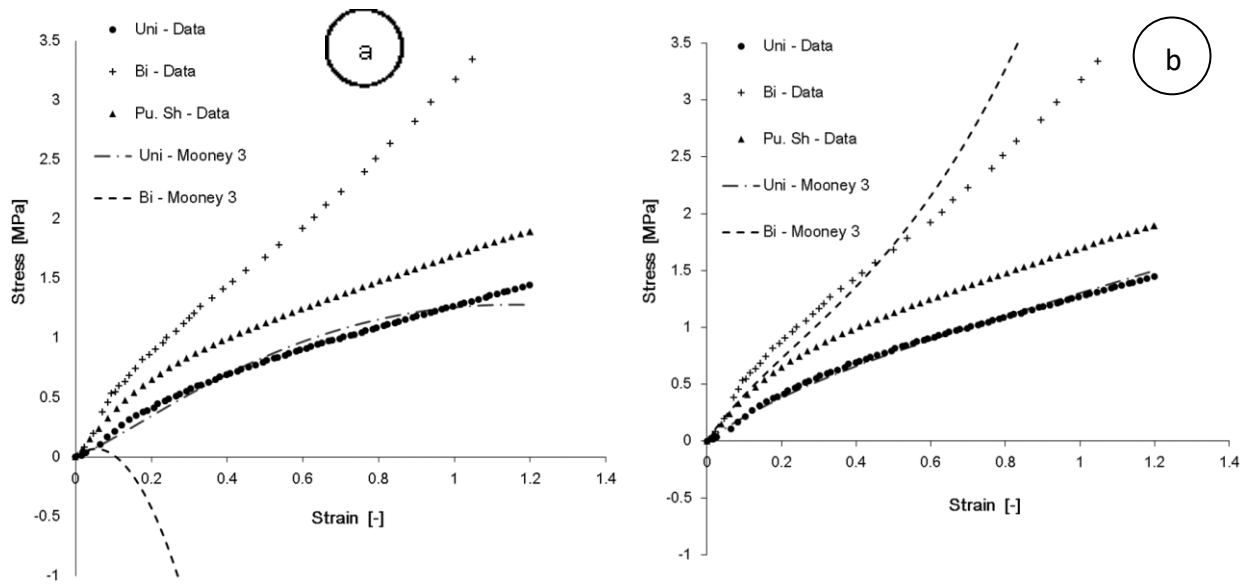


Fig. 4.2 Mooney three p. model comparison (a-only uniaxial, b-combined)

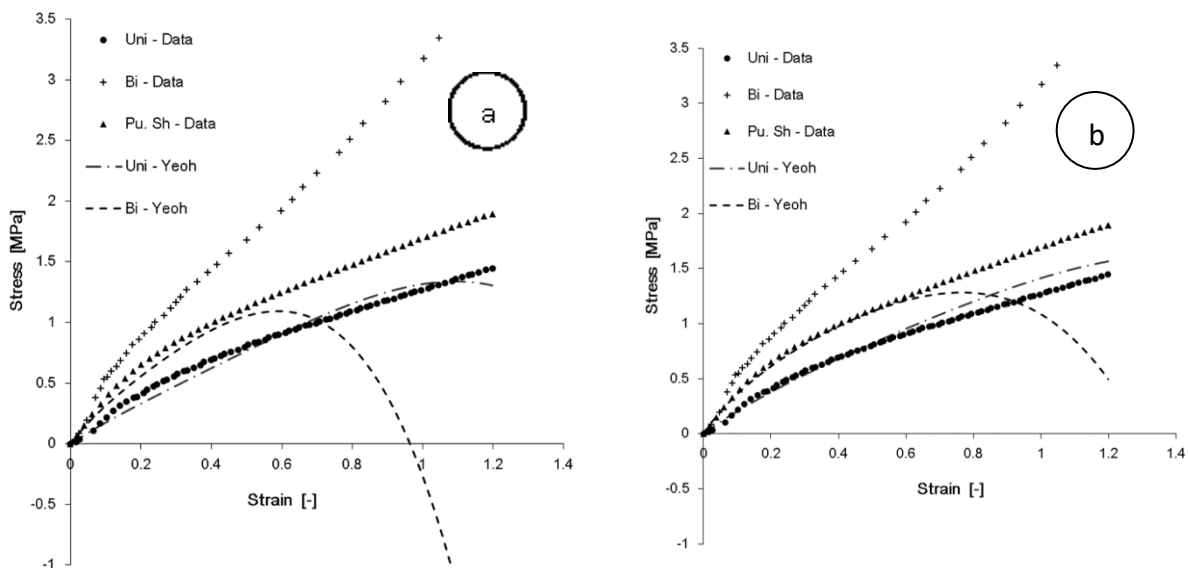


Fig. 4.3 Yeoh model comparison (a-only uniaxial, b-combined)

4.2 Initial Attempt in Solving the Problem (Exp. -2)

It was proven from first experiment that one data set used alone in data fitting, is not sufficient to obtain accurate results for material constants and thereby, for the mechanical characterization of rubber like materials. The status quo of the problem is as such, an effort was exerted in order to address the problem and several attempts were made to get a feasible solution to it. At this stage, a second experiment was done related to the topic and results of the experiment are presented here.

In this experiment, a set of stress-strain data was collected using uniaxial tension upon SBR rubber samples and it was then manipulated using a simple mathematical formula to get a hypothetical second data set. The equation used here is given in 4.1.

$$y_b = e^{ay_u} \tag{4.1}$$

In the formula given, y_b is the generated biaxial stress while y_u is the corresponding uniaxial stress. Factor a is used with three values, 0.6, 0.7 and 0.8. Generated data is given in fig. 4.4.a. After a close visual inspection, data set that resembles most to the original data set, 0.8 set was selected. With that data set as biaxial data, combined data fitting was done and Fig. 4.4.b gives the results.

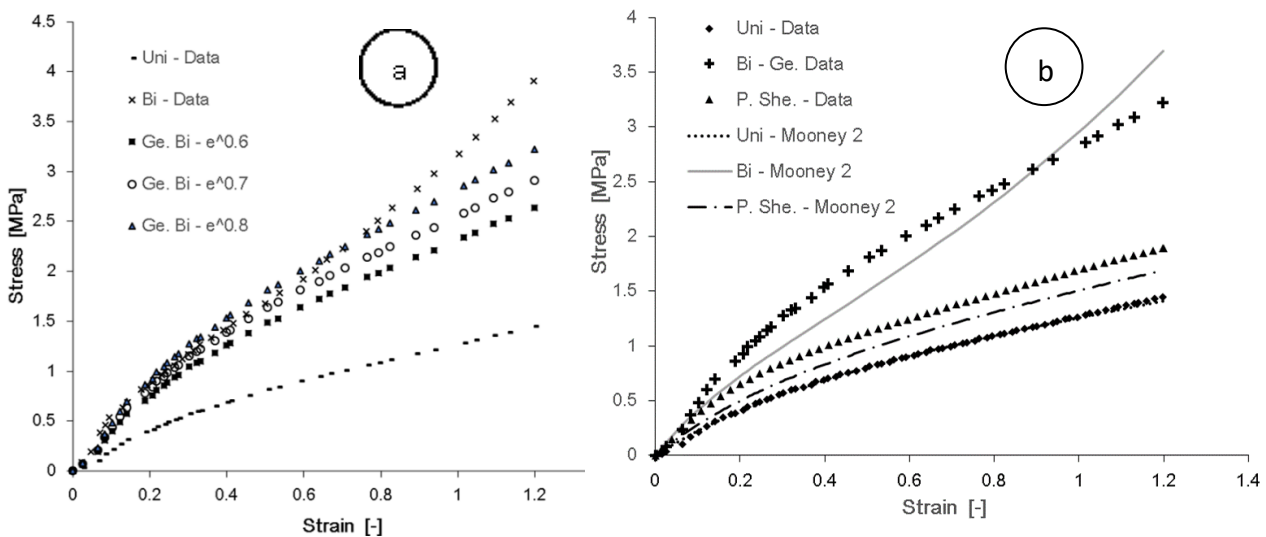


Fig. 4.4. a Comparison of generated biaxial data **Fig. 4.4. b** Combined data fitting with generated data.

In order to ascertain the improvement due to newly adopted method, a comparison was done between combine data fitting which utilized the data of uniaxial and newly generated biaxial and single data set fitting, i.e. uniaxial data, of results previously given in figure 4.1 (a).

Residue analysis was also done as to evaluate the data fitting. Resultant values are given in table 4.1.

Table 4.1. Residue Error values for two cases.

Curve type	R.S.S. for only uniaxial data fitting (Fig. 5.1 (a))	R.S.S. for combined data fitting (Exp. uniaxial and gen. biaxial data) (Fig. 5.5)
Uniaxial	1.7155	3.1275
Biaxial	32.025	1.1595
Pure shear	8.2099	1.6638

Fom the outcomes of this experiment therefore, clear improvement in fitting results could be observed both visually and statistically compared to only uniaxial data fitting.

4.3 An Improvement to the Initial Solution (Exp. -3)

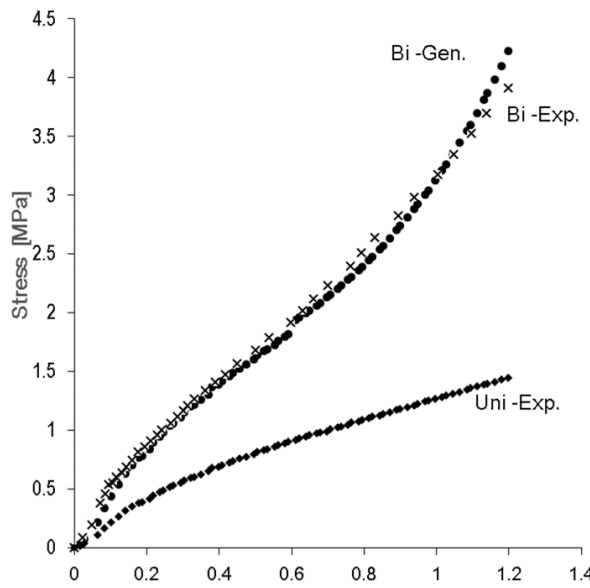


Fig. 4.5 Gen. Bi. Data and Exp. Bi. Data with Uni. data

Since there had been some discrepancies in the generated data set compared to real biaxial data in the previous experiment, further refinement of formula was needed. In order to improve results, some changes were done. The method which was adopted for the refinement of data is briefly given here.

In this experiment, uniaxial data set was separated in to two segments. Consequently, they were treated separately with two different formulas. With the use of two new equations, the biaxial stress data was generated. Two formulas used for the purpose is given

in equations 4.2 and 4.3.

$$\text{For } X < 0.6, \quad Y_b = e^{0.7} \times y_u \quad 4.2$$

$$\text{For } X > \text{ or } = 0.6, \quad Y_b = y_{u=0.6} + e^{2 \times (x_u - 0.6)} \quad 4.3$$

The data set obtained through the method then plotted in a graph alongside real biaxial and uniaxial data in order to examine the compatibility (Fig. 4.5).

Furthermore, data fitting was done separately with four different models, Mooney 2, Mooney 3, Yeoh, and Ogden to investigate the success of the method. Figures 4.6 to 4.9 give results of this comparison. Uniaxial, Biaxial and pure shear curves are named in these graphs as U, B and P respectively.

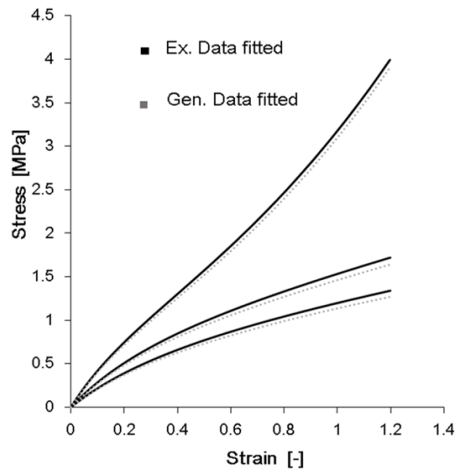
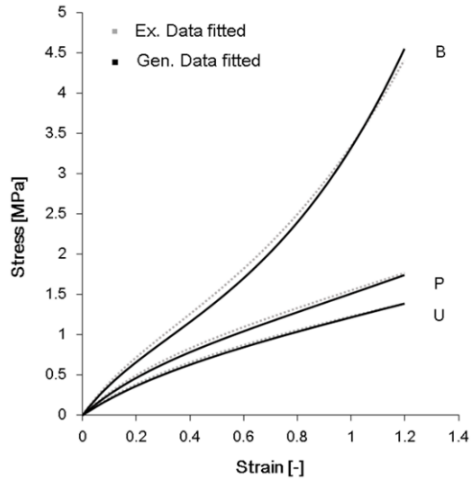


Fig. 4.6 Mooney-2 model comparison. **Fig. 4.7** Mooney 3 comparison

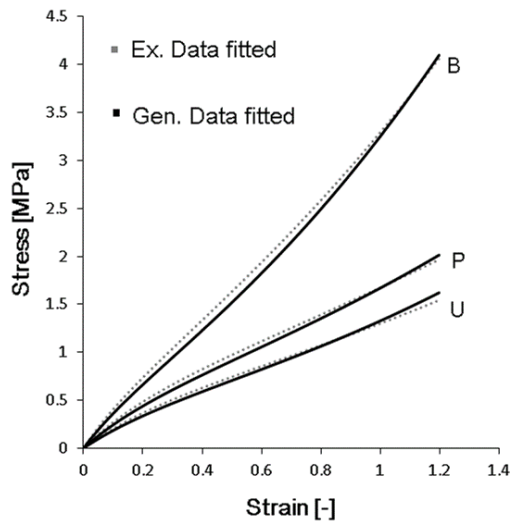
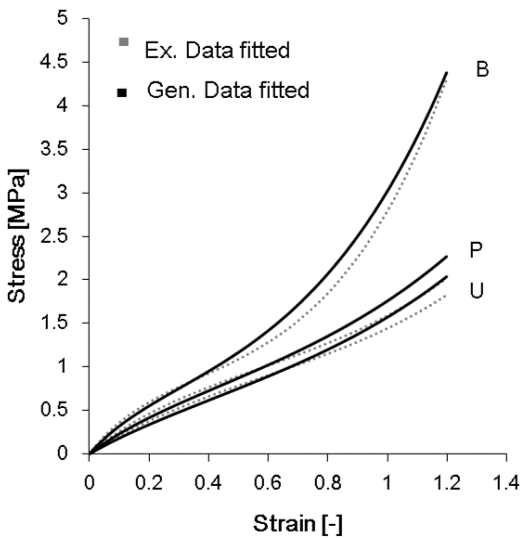


Fig. 4.8 Yeoh model comparison. **Fig. 4.9** Ogden model comparison

When consider these results, except for the Yeoh model, curves of all models show remarkable improvement related to the data fitting. Discrepancy showed by the Yeoh model might have been due to the fact that particular model is not suitable in representing this specific material characterization. Basically, it could be apportioned to a mismatch between the model and the material.

4.4 The Detailed Solution to the Problem (Final Experiments)

As to prove our approach in finding additional biaxial data set and thereby to establish the validity of the method, a set of experiments were done. There are six different types of material, M1 to M6, which is to be tested. The experiments which were planned here consisted of 30 uniaxial experiments and 10 biaxial experiments for each material. Therefore, given number of experiments were planned and carried out for each of them. In this section, the results of these experiments are discussed in detail.

Results of these experiments could be divided in to three segments. First section discusses the resultant data distribution related to both biaxial and uniaxial experiments. The next section would be dealing with the statistical analysis. Final and the last section discusses the fitted model curves related to real data and the generated data. Additional topic would be allocated to discuss the possibility of optimizing the solution.

Data Distribution Comparison

In order to get a relationship between uniaxial data and biaxial data, first of all, each material needed to be represented by single unique uniaxial and biaxial data set.

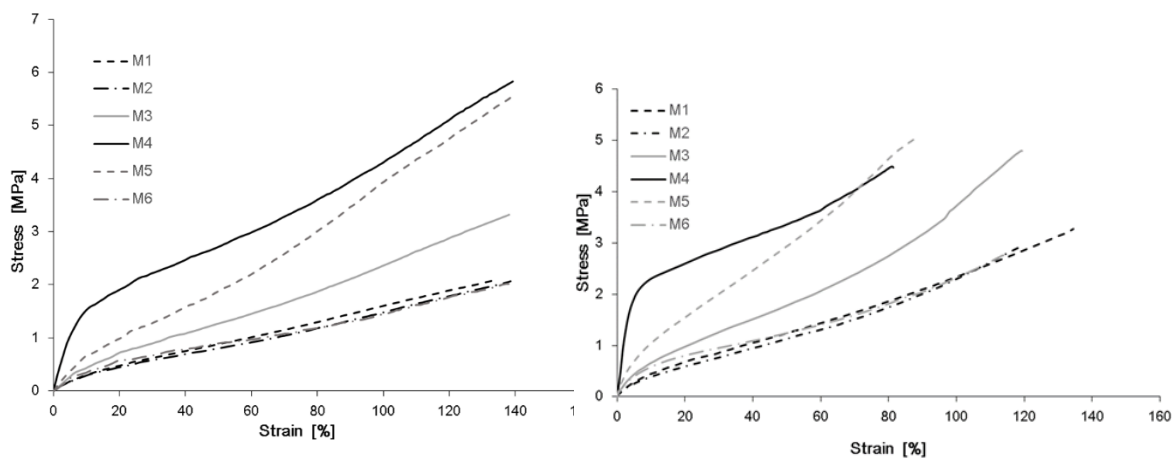


Fig. 4.10 Distribution of uniaxial data **Fig. 4.11** Distribution of biaxial data

As there were more than one set of data from each material, this was achieved by obtaining the average of multiple data sets. All thirty data sets were adjusted to have same number of data points and thereafter averaged data values were calculated by taking simple average with 30 number of points.

Averaged uniaxial data distributions of all materials are given in Fig.4.10. According to the figure, in general, all data sets depict typical uniaxial distributions. However, if we consider each distribution separately and observe closely, there are some minor differences. Two materials namely M4 and M5 stand out from others. They show unique distribution pattern. Furthermore, materials M1, M2 and M6 lies very close to each other. Position of material M3 is somewhat away from the rest. If we take the material M4, it is some out of general shape of biaxial data distribution. The trajectory in this case is unique and visibly has two portions to it. First segment appear to be rapidly increasing from zero up to around 10% strain. During the second half, data dispersion seems flattening.

By and large, all data sets seems having the sag in the middle potion. In some data sets, the middle portion sag is prominent while in others it is less dominant.

Distribution related to biaxial data for same six materials are given in figure 4.11. Initial visual inspection of this figure reveals a picture similar to the uniaxial data distributions previously discussed. However, in this case, differences between each individual set seems much dominant than previous instance.

In this case, all data distributions are shifted more towards stress axis as with the typical biaxial data distributions. Like in previous case, same three data sets are visibly separated from the rest. Though it might not be significant, two data sets, M4 and M5 are crossing each other at somewhere in the vicinity of 70% strain. Data set M4, change its trajectory in this case at around 6% strain which is little earlier than in previous uniaxial case. Out of two segments of this distribution, first segment seems steeper than in previous instance.

The Relationship between Uniaxial and Biaxial Data Distributions

In order to construct a relationship between uniaxial data and biaxial data, each data set must be representative average curve of the respective material. Therefore, in the previous section, these average curves were obtained for each material for both uniaxial and biaxial deformation modes. Considering the data distributions of these materials, and the relationship we already discussed, a mathematical formula which is given in equation 4.4 is arranged in order to link two data sets.

$$\sigma_b = \sigma_u \times e^x \quad 4.4$$

In the equation, σ_b is the biaxial stress at an arbitrary point in the stain axis while σ_u is the corresponding uniaxial stress at the same point. Exponent x of the exponential function is a positive real number.

Using this relationship, several biaxial data sets were calculated after assigning different values to the unknown number x . From these initial trials, it could be selected a suitable value for x somewhere near 0.4, considering the generated biaxial data and the actual biaxial data. After that, further improvements were done and x was fixed at 0.35.

With selection of particular value for x , using above equation, corresponding biaxial values could be generated for each uniaxial stress. Using this method, all biaxial stresses were calculated and plotted together with related uniaxial and experimental biaxial data, as given below from figures 4.12-4.17. Newly created substitute data for biaxial stress- strain distributions are, for identification purposes called hereafter as generated data whenever given in the text.

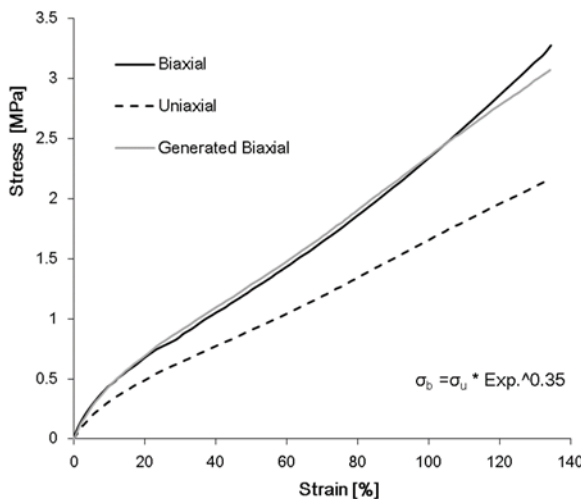


Fig. 4.12 Generated data for M1

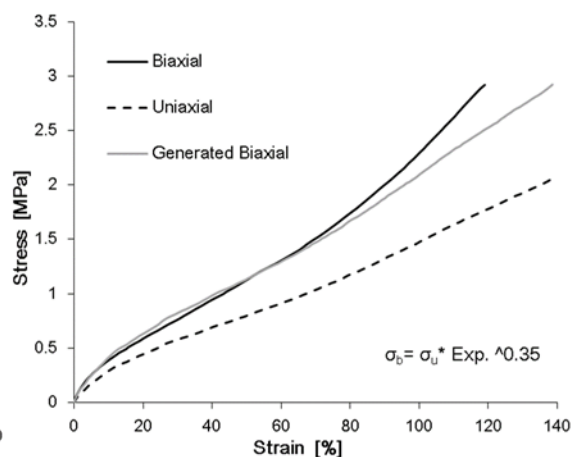


Fig. 4.13 Generated data for M2

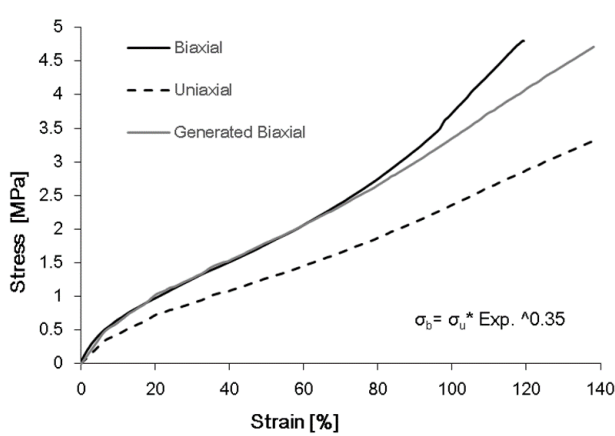


Fig. 4.14 Generated data for M3

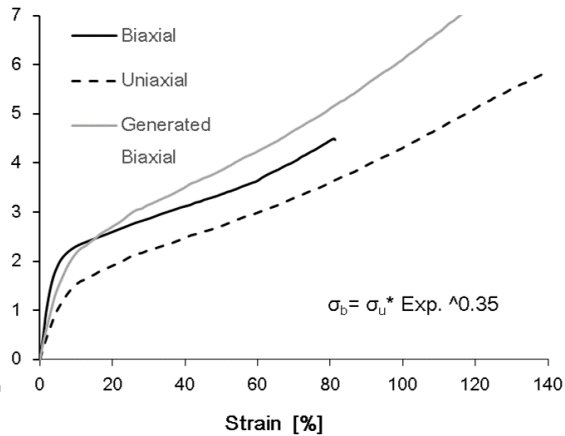


Fig. 4.15 Generated data for M4

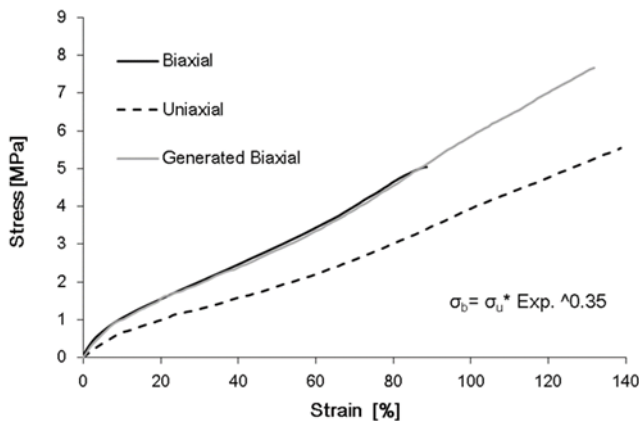


Fig. 4.16 Generated data for M5

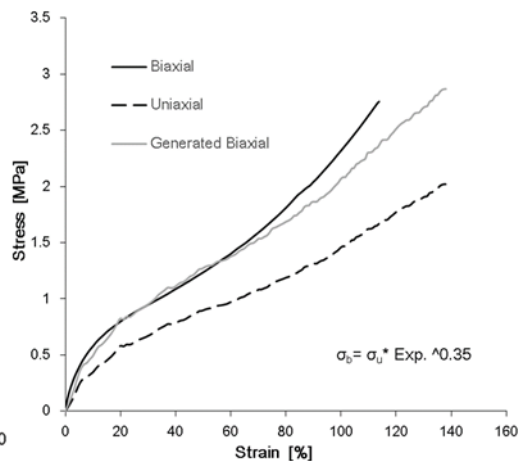


Fig. 4.17 Generated data for M6

As visible from these graphs, generated data sets are located in general, close proximity to the respective experimental data sets. Only exception is the M4 data set where generated data set is taking a trajectory similar to uniaxial and hence not showing close resemblance to the biaxial experimental data distribution. However, in this particular case, the said difference could be observed when compare uniaxial and biaxial experimental data sets as well. On the other hand, when we examine all generated curves closely, a certain deviation could be observed at the later part of strains for each material. These deviations are happened to be in various proportions according to each of these materials.

Out of all generated biaxial data sets related to these six materials, the data generated using uniaxial data of material 5 seems the best and the closest to the original data.

The Statistical Reasoning

After generating biaxial data distributions for each and every material tested, next task was to test statistically how close these generated data to the real data. In order to do verify this, we followed a typical significant test devised as to suite this particular case.

Significant testing is normally done in order to estimate the level of confidence with witch one can forecast the population from a sample. This is a typical statistical testing method and here, it is adopted to include this particular situation as follows.

According to the method discussed here, first of all averaged biaxial data set is divided in to five equal segments. This is done according to the stress obtained by dividing maximum average stress in to five (Fig. 4.18). Idea behind this effort is to get five different points in the data distribution to compare real and generated biaxial data.

Thereafter, corresponding strain values were selected. By plotting generated biaxial data distribution according to the material in the same graph, equivalent generated biaxial stress values for these strain tapping points could be calculated. At the same time, all experimental biaxial data were plotted in the same graph and equivalent stress values corresponding to each tapping point strain is also collected.

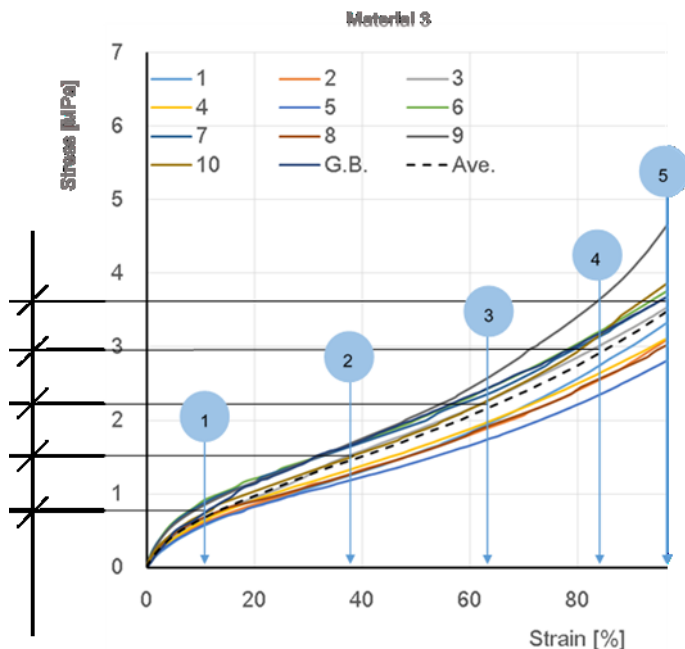


Fig. 4.18 five tapping points

For the demonstration purpose, the stress values obtained for material M1 is given in the table 4.2, below for the ten sample graphs and for that of generated graph. The first column of the table gives each tapping point strains collected through average curve maximum stress dividing in to five.

Table 4. 2 Stress values for five tapping points for M1

Tap. Pt. strain [%]	Curve number & corresponding stress [Mpa]										Gen. curve Stress [MPa]
	1	2	3	4	5	6	7	8	9	10	
11.53	0.68	0.54	0.52	0.52	0.48	0.51	0.42	0.56	0.46	0.57	0.49
38.85	1.21	1.18	0.99	1.05	1.00	1.02	0.76	1.08	0.93	1.17	1.07
63.51	1.71	1.71	1.42	1.50	1.46	1.48	1.14	1.54	1.36	1.68	1.55
87.83	2.22	2.31	1.91	2.01	1.98	2.01	1.84	2.04	1.86	2.30	2.08
108.43	2.70	2.87	2.37	2.50	2.47	2.51	2.37	2.52	2.35	2.95	2.53

Ten stress data values were examined against respective generated stress value using statistical significant test. For the statistical significant test or t-test, null and alternative hypothesis arguments were constructed as follows.

Null Hypothesis

$$H_0 : \mu = \bar{x} \quad 4.5$$

Alternative Hypothesis

$$H_1 : \mu \neq \bar{x} \quad 4.6$$

Where, μ is the Generated stress while \bar{x} is the mean of ten values of ten curves at the same taping point. Results of this significant test are tabulated here. (Table 4.3)

Table. 4.3 p values derived using student's t -distribution

Material	Stain point & value [%]	p value
M1	1. 11.53	0.095
	2. 38.85	0.454
	3. 63.50	0.378
	4. 87.83	0.551
	5. 108.43	0.676
M2	1. 9.32	0.389
	2. 27.82	0.113
	3. 49.88	0.197
	4. 69.16	0.861
	5. 85.15	0.178
M3	1. 7.21	0.179
	2. 25.67	0.522
	3. 47.94	0.391
	4. 67.32	0.951
	5. 83.29	0.425
M4	1. 0.94	0.016
	2. 3.08	0.001
	3. 8.76	0.001
	4. 37.94	0.000
	5. 65.42	0.000
M5	1. 4.04	0.000
	2. 16.90	0.041
	3. 34.07	0.462
	4. 49.88	0.086
	5. 64.35	0.257
M6	1. 4.04	0.001
	2. 15.79	0.059
	3. 39.77	0.676
	4. 62.66	0.929
	5. 79.90	0.227

By referring the third column of the table, we can see that critical p values for most of the significant tests are above 0.05. This means that the forecast can't be rejected or in other words, with 95 percent confidence we can say that null hypothesis cannot be rejected in such cases.

However, there are few exceptions where, the p value is less than 0.05. In particular, for the material M4, with relation to all points of concern, shows such

low values. As we have already mentioned, this particular material seems deviating from others and behaving differently from the group. Therefore, such a result could be expected for the statistical test as well.

In other cases where P value is less than 0.05, it would be appropriate to lower the confidence interval and check the results once again. This suggestion could be justified as typical behavior of rubber materials allow such wide margins in the testing.

The Generated Biaxial Data Set Optimization

As part of the generalized biaxial generating formula is an exponential function, the exponent can take many values. The value of exponent affect the relative position of the generated data set. Therefore, we examine the possibility of getting a unique value for the exponent which would provide the optimal position for the data set related each material. At the end, further examination was done as to find one optimal value representing whole material group.

The method used here is explained as follows. Considering the five tapping points used for extraction of data for statistical calculation, by referring figure 4.19, in order to minimize the error between biaxial experimental data and generated data (Δy), following formula could be drawn (Eq. 4.7-4.9).

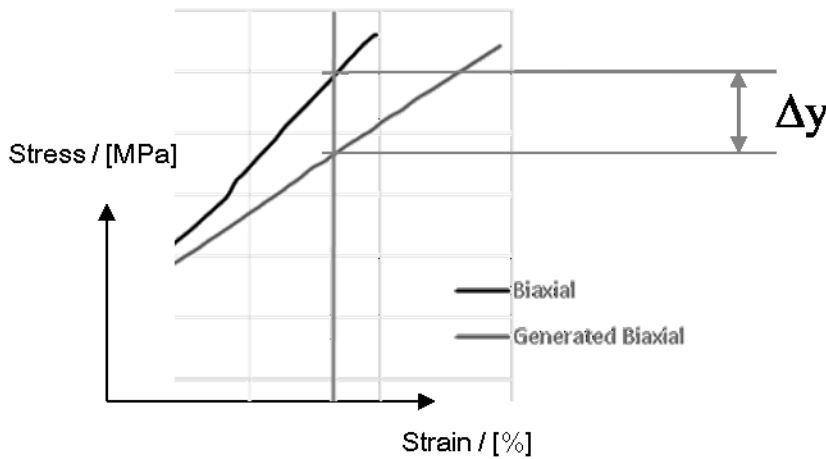


Fig. 4.19 Error between real and generated biaxial data

$$f(x) = \sum_1^5 \left[(y_{bi} - y_{gbi})^2 \right] \quad 4.7$$

$$f(x) = \sum_1^5 \left[(y_{bi} - y_{ui} \times e^x)^2 \right] \quad 4.8$$

$$f(x) = \sum_1^5 y_{bi}^2 - 2 \times \sum_1^5 y_{bi} \times y_{ui} \times e^x + \sum_1^5 y_{ui}^2 \times e^{2x} \quad 4.9$$

Using this formula, by allocating values to exponent x , from 0.005 to 0.7, $f(x)$ vs x was drawn. Graph drawn for material M1 to M6 are given below from Fig. 4.20 to 4.25 looks as follows. From these graphs, it is possible to find the value of exponent x where, error of real biaxial data set and generated data is minimal.

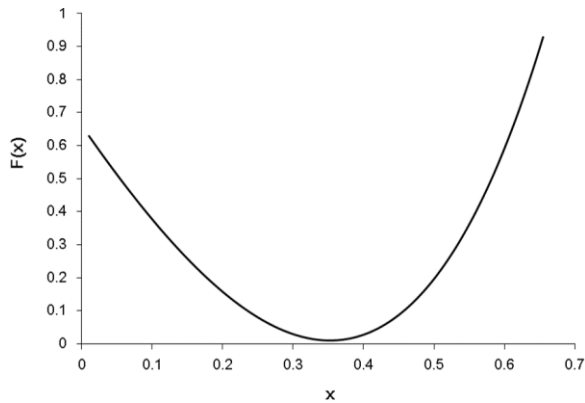


Fig. 4.20 Error calculation for M1

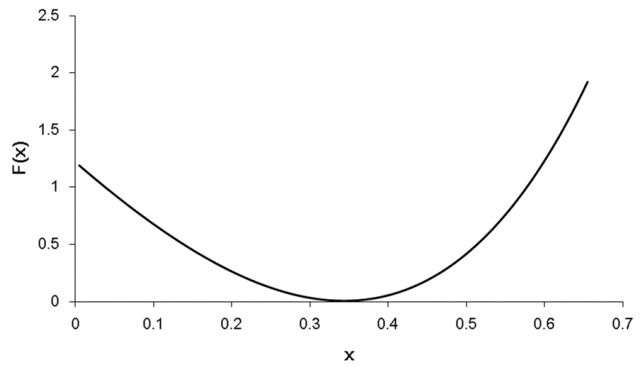


Fig. 4.21 Error calculation for M2

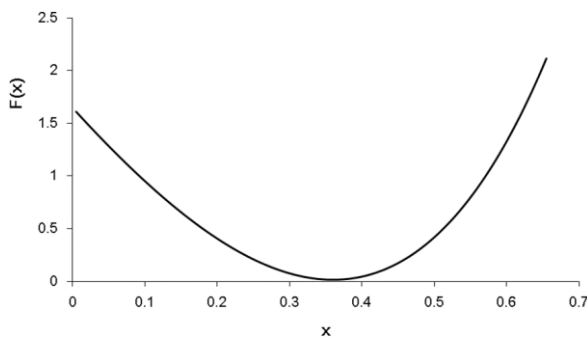


Fig. 4.22 Error calculation for M3

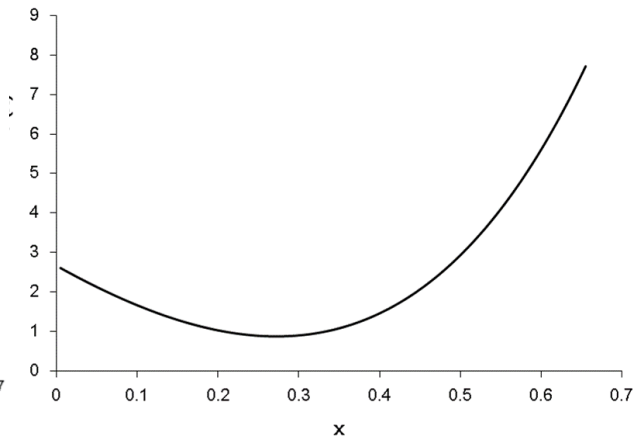


Fig. 4.23 Error calculation for M4

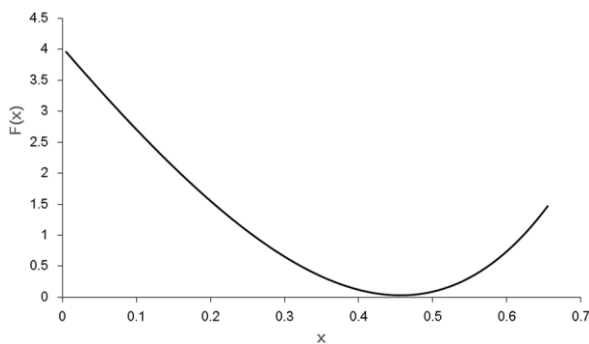


Fig. 4.24 Error calculation for M5

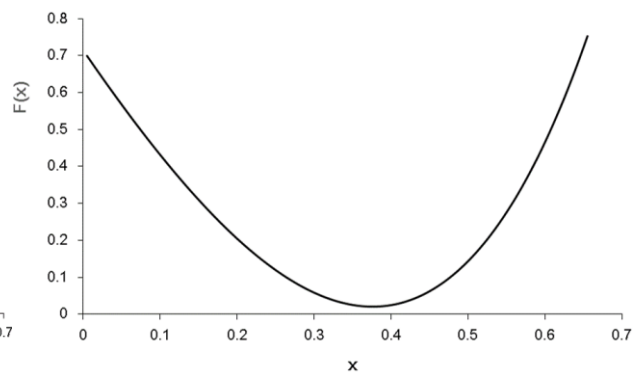


Fig. 4.25 Error calculation for M6

From these graphs, it can be evaluated the optimal value of x for materials M1 to M6 as follows. (Table 4.4)

Table 4.4 Optimal values for Exponent.

Material	Optimal Value
M1	0.35
M2	0.35
M3	0.36
M4	0.27
M5	0.46
M6	0.38

Furthermore, considering all the data together, it is possible to get a one exponent for all materials. Idea is to get a one formula for the material group. For this task, calculations were done and related plot was presented in Fig. 4.26.

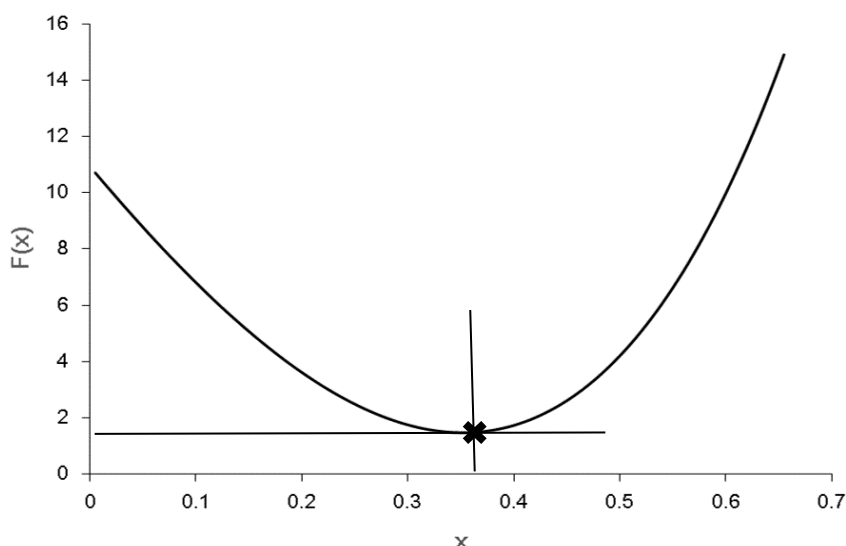


Fig. 4.26 Error calculation for all materials together

Considering the overall graph, optimal value for this material group could be fixed at 0.3525.

The model testing for the newly generated biaxial data

Having examined and obtained positive results for the statistical testing, next and the last step was to see how these artificial data would work with some of the common models when used for combined data fitting with uniaxial data. In this section of the work, several models were examined for the compatibility and to

obtain the correct model to represent the material group. However, some of the material models tested were not compatible with this material group and showed inconsistency results. Mooney two parameter model was the most compatible. Results of this model fitting for six materials separately are given from figures 4.27 to 4.32.

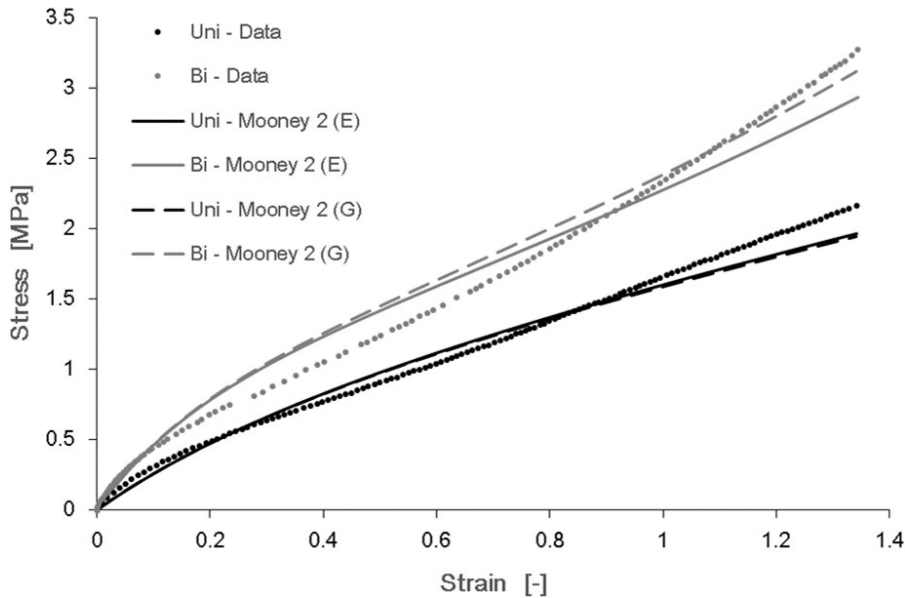


Fig. 4.27 Mooney-2 model: Generated and real data comparison – M1

Combined data fitting of both real biaxial data with uniaxial data and generated biaxial data with uniaxial data were done separately. Uniaxial model seems coinciding in these two instances (Fig. 4.27).

Data fitting results given in for material 2 in figure 4.28, also shows similar results like in previous case. However, unlike previous instance, in these cases, data and the model curves seems somewhat compatible with each other.

Material 3 data fitting results given in figure 4.29 also gives a set of curves somewhat similar to M2 curves, though with higher stresses.

As it was the case with M4, the model curves seems deviating from the data distributions (Fig. 4.30). On the other hand, two combined data fitted modal curves are deviating from each other from early strain value

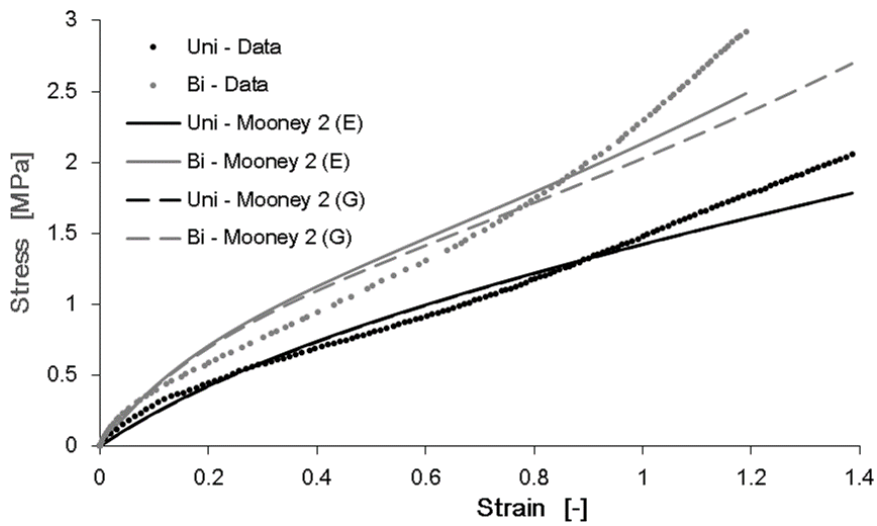


Fig. 4.28 Mooney-2 model: Generated and real data comparison – M2

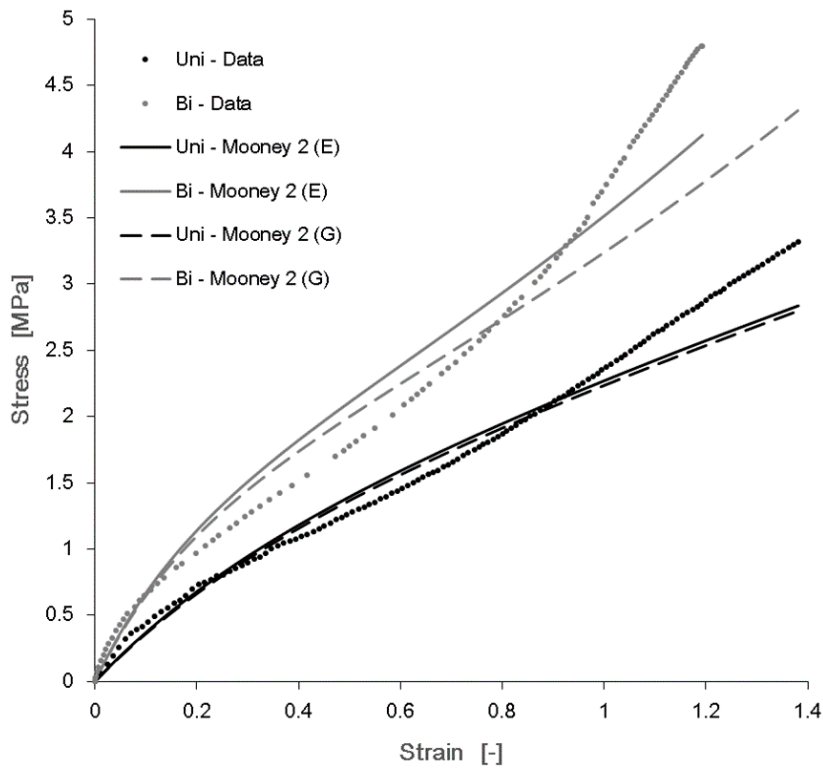


Fig. 4.29 Mooney-2 model: Generated and real data comparison – M3

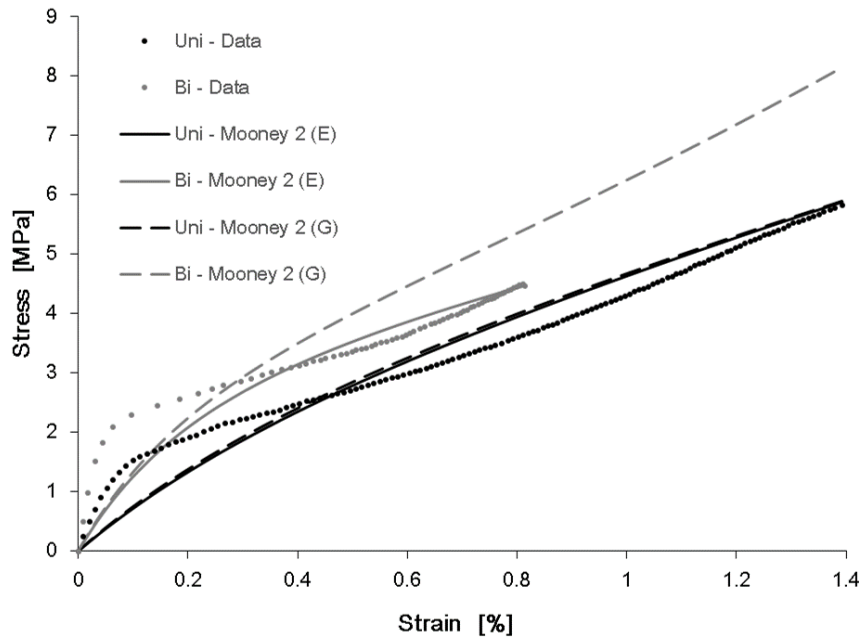


Fig. 4.30 Mooney-2 model: Generated and real data comparison – M4

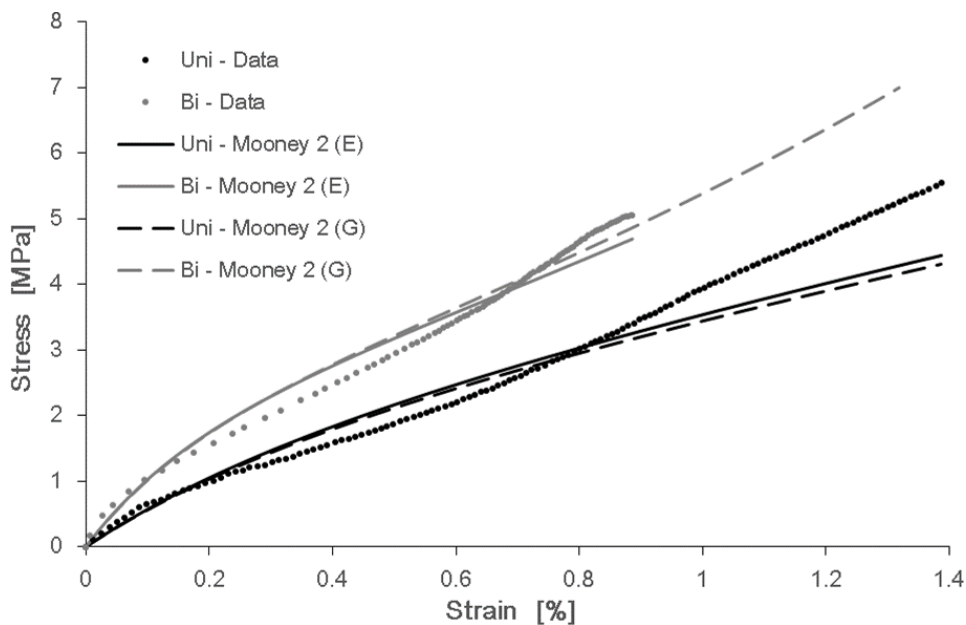


Fig. 4.31 Mooney-2 model: Generated and real data comparison – M5

For the material M5, (Fig. 4.31) once again, biaxial and generated biaxial model curves are close to each other, but data set seems away from the models. In the case of uniaxial this is very much improved and all three are laying near to each other. Stresses are relatively high in this case.

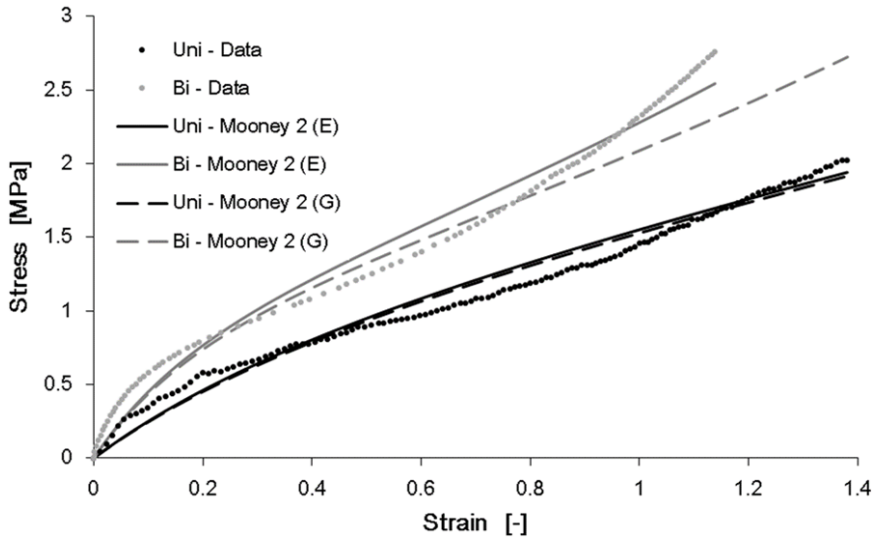


Fig. 4.32 Mooney-2 model: Generated and real data comparison – M6

In the case of M6, (Fig. 4.32) model curves are crammed between two data sets. This means, model is lower in the biaxial and higher in the uniaxial when it comes to represent respective data. However, for both instances, real and generated data fitted models give somewhat similar trajectories, though in biaxial case, at later part the generated curve is little off-shoot from the real one.

Altogether, data fitting for Mooney-2 model shows mixed results for six materials discussed. As these materials are different from one another in the way of hardness and carbon black content which consequently leads to property change, such variations could be expected.

5. CONCLUSION

The research work included in this thesis touched the area of hyperelastic material characterization. As materials of this nature are not the easiest to characterize, the only way of doing it is by fitting data to an appropriate model. There are many Models to select with. However, in order to get material constants, due to lack of enough data sets, usually one data set was selected for fitting. This practice proved to be erroneous. Therefore, effort was taken herein to solve this problem by generating an additional data set whenever one data set is available.

To begin with, error in the data fitting with one data set was established. In this effort, two data fittings, only uniaxial and combined uniaxial plus biaxial were done and results were compared. Biaxial data and curves were not in any way matching in the case of single data fitting, where as in combined fitting, curves were nicely seen near to respective data. With that results, it was proved the erroneous behaviour of the method of single data fitting.

Having done that, first proposal was given in the way of exponential function as to obtain a second data set from the available uniaxial data. The outcome of first effort showed promising results in the way of secondary data. Second data set obtained through the method was consequently used together with uniaxial data for combine data fitting. Clear improvement in fitting results could be observed both visually and statistically, compared to only uniaxial data fitting. Possibility of further improvement to the method of generating of secondary data was examined. In this method, uniaxial data was divided in two segments and each segment was separately addressed with different empirical formula to get the second data set. Final results of this experiment showed a further improvement to the overall fitting results. However, this method creates some complexities to the solution and therefore, was not further tested for application possibility.

Thereafter, final experiments were done using six different materials. Once again, exponential function was used with a different exponent. This time, overall results were encouraging and therefore continued with further testing. Out of six materials, five were reasonably successful. Only material M4 showed some resentment to the method. Later on, a statistical confidence interval test was done in order to check the closeness of two biaxial data dispersions. Students T table was used for this examination. Consequently, a combined data fitting also was done with Mooney 2 model and results thus obtained once again were indicating to the useful nature of the method. However, Material M4 showed some incompatibilities in data fitting results as well.

Finally, exact exponent was searched for to use in exponential function in each material case. Certain type of least square method was used here. In this search, it was found out that most of the time exponent value oscillate as around 0.35. When method used with all material data together, value came to stand at 0.355 exact figure.

CONTRIBUTION TO SCIENCE AND PRACTICE

The thesis aimed at providing a solution to a problematic situation that arises when there is not sufficient data sets available for data fitting in the mechanical characterization of hyperelastic materials. By providing secondary data set which resembles biaxial data, the problem could be overcome with reasonably accurate results. Such solution of the problem, contributed to the science in following manner.

- Method could improve the results by eliminating erroneous practice of single data set fitting.
- This is a cost-effective method of mechanical characterization of hyperelastic materials.
- The method reduces aggregate time consumed for characterization by way of additional experiments.
- Eliminate inaccuracies attached with biaxial testing as method nullifies the requirement for additional tests.
- The method open up a new area of research to find extra pure shear data set with similar type of method
- At the same time, method could be further developed to accommodate many more materials.

ACKNOWLEDGEMENT

The task of writing down any thesis and undertaking the related work is not certainly an individual effort, though there is a principle candidate. Mine is not an exception. There're many who helped me along the way. Therefore, it is my duty to thank them all.

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LIST OF SYMBOLS AND ACRONYMS

a	-	Factor	-
b, n	-	Factors	-
e	-	Deformation measure	-
e'	-	Error value	-
μ	-	Generated stress	MPa
y_b	-	Generated biaxial data	MPa
y_u	-	Uniaxial stress	MPa
σ_b	-	Biaxial stress	MPa
σ_u	-	Uniaxial stress	MPa

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